

RANDOM VIBRATION ANALYSIS  
OF SPACE FLIGHT HARDWARE USING NASTRAN

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During liftoff and ascent flight phases, the Space Transportation System (STS) and payloads are exposed to the random acoustic environment produced by engine exhaust plumes and aerodynamic disturbances. The analysis of payloads for randomly fluctuating loads is usually carried out using the Miles' relationship. This approximation technique computes an equivalent load factor as a function of the natural frequency of the structure, the power spectral density of the excitation, and the magnification factor at resonance. Due to the assumptions inherent in Miles' equation, random load factors are often overestimated by this approach. In such cases, the estimates can be refined using alternate techniques such as time domain simulations or frequency domain spectral analysis. This paper describes the use of NASTRAN to compute more realistic random load factors through spectral analysis. The procedure is illustrated using Spacelab Life Sciences (SLS-1) payloads and certain unique features of this problem are described. The solutions are compared with Miles' results in order to establish trends at over or under prediction.

## INTRODUCTION

During the past decade, the U.S. Spacelab program has made significant contributions to the advancement of space exploration and research. The Spacelab is a reusable laboratory that is carried in the cargo bay of the Space Shuttle. Experiments in several different disciplines such as astronomy, life sciences, and material science are accommodated in this modular laboratory for various shuttle missions. The module also contains utilities, computers, and work areas to support the experiments. The experiment hardware is mounted in instrument racks located on either side of the module, in overhead lockers, or in the center aisle, as shown in Figure 1.

During liftoff and ascent flight events, the Shuttle and its payload are exposed to the acoustic environment produced by engine exhaust plumes and aerodynamic disturbances. Random vibrations are created by the response of the module shell to the acoustic noise inside the cargo bay. The vibrations of the shell are transmitted through the support structures (racks, mounting frames, etc.) to the payload equipment. The vibration levels that the equipment has to withstand depend on its own dynamic characteristics and its location inside the Spacelab. The equipment and its structural interfaces must be analyzed for these random loads in order to ensure the integrity and flight worthiness of the system.

The analysis of flight hardware for random loads often relies on approximate formulations like the Miles' relation (ref. 1) to generate limit load factors for structural design. Due to the assumptions inherent in Miles' equation, the random vibration criteria developed through this approach tend to be over-conservative. In such cases, the results can be refined using alternate analysis techniques like time domain simulations or frequency domain spectral analysis. This paper describes the use of NASTRAN to perform spectral analysis to establish more realistic design loads. The procedure is illustrated using the Neck Chamber Pressure System (NCPS) assembly which will be flown on the Spacelab Life Sciences (SLS-1) mission.

## ANALYSIS BASED ON MILES' EQUATION

For a lightly damped single-degree-of-freedom (SDOF) oscillator subjected to random excitation through its base motion, Miles' relation is used as follows.

$$N_r = 3 \sqrt{\frac{\pi}{2}} f_n A Q \quad (1)$$

$N_r$  is the limit random load factor (g units),  $f_n$  is the resonance frequency (Hz) of the SDOF system,  $A$  is the power spectral density ( $g^2/Hz$ ) of base acceleration at the resonance frequency, and  $Q$  is the dynamic magnification factor at

resonance. The quantity under the square root represents the mean square acceleration response and the factor 3 indicates the 3-sigma probability of occurrence, i.e., the probability of exceeding  $N_r$  is 0.26%.

The mean square response is the area under the response spectral density curve and is given by

$$\langle u^2 \rangle = \int S(\omega) |H(\omega)|^2 d\omega \quad (2)$$

where  $H(\omega)$  is the transfer function of the system,  $S(\omega)$  is the base acceleration spectral density function, and  $\omega$  is the frequency in radians. The derivation of Miles' relation is based on the following simplifying assumptions for evaluating the integral in Eqn (2).

- 1) The actual spectral density of base excitation,  $S(\omega)$ , is a slowly varying function in the vicinity of resonance. It can be conveniently approximated by a constant or white-noise spectral density,  $A = S(\omega_n)$ , for computing the mean square response.
- 2) Only the excitation energy contained within the system's bandwidth is transmitted. The rest is filtered away by the system and does not contribute to the mean square response.

These assumptions are valid in the case of lightly damped systems with damping factor  $\xi \ll 1$ . For such systems, the function  $|H(\omega)|^2$  is very sharply peaked at  $\omega = \omega_n$ , and reduces to half its peak value at a short distance,  $2\xi\omega_n$ , on either side of the peak. This distance, called the half power bandwidth, is very narrow for lightly damped systems. With the assumptions mentioned above, the integral in Eqn. 2 can be approximated by a rectangular area with base equal to the bandwidth and height equal to the product of the constant value of excitation spectral density and the peak value of transmittancy. This gives

$$\langle u^2 \rangle \simeq \frac{\pi}{2} f_n A Q \quad (3)$$

from which Miles' relation follows.

In order to use Miles' relation for the analysis of flight hardware, the natural frequency of the equipment is first determined through analysis or test. The input random excitation spectrum for the equipment is then determined as a function of its location, its mounting configuration, and its mass. The input excitation spectrum has been established using data from previous flight or ground tests and is provided in the Spacelab Payload Accommodation Handbook (ref. 2). The spectral density at the resonance frequency of the component is found from this data. The dynamic magnification factor at resonance  $Q = 1/2\xi$ , is indicative of the system damping and is determined experimentally. For

components mounted on isolators,  $Q$  is determined from the manufacturer's data on the isolator mounting. These values are substituted in Eqn (1) to obtain the design random load factor,  $N_r$ . As the random vibration environment occurs simultaneously with other load environments during various mission phases, the estimated values of  $N_r$  are combined with the appropriate quasi-static, thermal, pressure, and crew-induced load factors to generate design load cases for component analysis.

Due to the assumptions inherent in Miles' relation and the idealization of the component as a single-degree-of-freedom resonator, the computed random load factors will be approximate. They tend to be overly conservative, especially when the natural frequency of the system is close to the peak frequency of the excitation spectrum. When the predicted random loads are unreasonably high, they lead to difficult design problems and alternate approaches are necessary to refine the random load estimates.

## ANALYSIS BASED ON SPECTRAL ANALYSIS

The dynamic behaviour of large structural/mechanical systems can be adequately predicted only by multi-degree-of-freedom (MDOF) models. For linear MDOF systems, the dynamic characteristics are specified by a matrix of transfer functions,  $[H]$ , whose elements  $H_{jk}$  represent the ratio of steady-state response at point  $j$  to a sinusoidal excitation at point  $k$ . For displacement response

$$[H(\omega)] = [-[M] \omega^2 + i [B] \omega + [K]]^{-1} \quad (4)$$

where  $[M]$ ,  $[B]$ , and  $[K]$  represent the mass, damping, and stiffness matrices of the discrete model and  $\omega$  is the excitation frequency.

The response of linear MDOF systems subjected to random excitation can be computed using well-established spectral analysis techniques. According to the theory of random vibrations, the response of a linear system with transfer function  $[H(\omega)]$ , subjected to a stationary random load  $\{P(t)\}$ , is given by

$$[S_{uu}(\omega)] = [H(\omega)] [S_{pp}(\omega)] [H'(\omega)]^T \quad (5)$$

where  $[S_{uu}(\omega)]$  and  $[S_{pp}(\omega)]$  are the matrices of response and excitation spectral density functions and  $*$  and  $T$  represent the complex conjugate and transpose operations, respectively. These matrices will have real auto-spectral density functions as their diagonal elements and complex cross-spectral density functions as their off-diagonal elements. By integrating the area under the response spectral density curve, the mean square response at any nodal point in the model can be obtained. The foregoing development for mean-square displacement response can be generalized to provide mean-square values for other response quantities such as velocity, acceleration, or stress. It is only

necessary to replace the transfer function matrix for displacement by the corresponding transfer function matrix for the desired response. The generalization also applies to the excitation which may be a point force, a loading condition (i.e., an ensemble of applied forces that are perfectly correlated), or enforced motion.

The analysis of flight hardware subjected to random excitation can be carried out using the spectral analysis features of NASTRAN. A finite element model of the component is created which can accurately represent all its dominant modes in the excitation frequency range. The response calculations are carried out in two separate functional modules. First, the transfer function of the system corresponding to the desired response is computed in the Frequency Response module and then, the power spectral densities and other response statistics are computed in the Random Analysis module. The direct or modal superposition approaches can be used to perform the frequency response analysis. For each excitation source,  $p_k$ , the nodal response,  $u_j$ , is determined at a series of user specified frequencies,  $\omega_i$ . The ratio of output to input represents the transfer function element,  $H_{jk}(\omega_i)$ . This is determined for each excitation source and the transfer function matrix,  $[H]$ , is assembled from the results.

In NASTRAN, random vibration analysis is treated as a data reduction procedure that is applied to the results of frequency response analysis. The inputs to the random analysis module are the frequency responses of desired output quantities due to different load sources and the auto- and cross-spectral densities of these random load sources. Each load source is referred to by a separate subcase in the case control deck and their spectral densities are specified as tabular functions of frequency in bulk data cards. If the sources are statistically uncorrelated, only the auto-spectral densities need be defined. The power spectral densities of response are calculated using Eqn (5) and the root-mean-square (rms) response is evaluated by numerically integrating the area under the spectral density curve. The results are printed and plotted for specified degrees of freedom of the model.

As mentioned earlier, the random excitation applied to the structure could be a force, an enforced motion, or some other general form of excitation. In the case of Spacelab payloads, the random excitation is specified in terms of an acceleration spectrum applied at the structural support points. The "large mass" approach may be used to simulate this loading condition. This involves lumping a fictitious large mass,  $M_a$ , at the degree of freedom in which the acceleration is to be enforced. An applied force equal to  $M_a$  times the required acceleration is also prescribed for that degree of freedom. The inertia force is made so dominant through this operation that the resulting acceleration is very close to the required value.

## ILLUSTRATIVE EXAMPLE

The development of random vibration load factors for flight hardware is illustrated in this section using the Neck Chamber Pressure System assembly (NCPS). This Life Sciences Laboratory equipment item is used to study the effect of weightlessness on human cardiovascular control mechanisms and will be flown on a future Space Shuttle mission. The NCPS assembly is composed of an experiment enclosure which houses several components including a central processing unit, a motor control unit, two motor driven bellows, and a pressure gauge (Fig. 2). The assembly is mounted in an experiment rack using two support rails which are attached to the front and rear rack posts on either side. The front panel of the enclosure is bolted to the front rack post flanges at eight locations. The whole assembly weighs 48 pounds, and the installation kit including the slides, fittings, and fasteners weighs an additional five pounds.

A finite element model of the NCPS assembly is constructed using mostly plate (CQUAD2 and CTRIA2) and bar (CBAR) elements. The model has a total of 206 grid points and 192 structural elements. The masses of internal components are lumped at the respective centers of gravity, and stiff bar elements are used to connect them to the attachment points. The fasteners are modelled using rigid elements. Eigenanalysis was performed on the model with free boundary conditions to verify that the model has six rigid body modes. The analysis is repeated, with the rack-to-component interface points appropriately constrained, in order to determine the flexible modes of the component. The first twenty frequencies of the constrained model are shown in Table 1. An inspection of the modal deformation plots and mass participation factors shows that the first system mode in X direction is 80 Hz. The power spectral density of the input excitation corresponding to this frequency is found to be 0.02 g<sup>2</sup>/Hz from Figure 3. For a conservative estimate of Q = 10 for the dynamic magnification factor, Miles' relation (Eqn 1) yields

$$N_{rx} = 15.04 \text{ g units} \quad (6)$$

Random load factors,  $N_{ry}$  and  $N_{rz}$ , are computed in a similar manner.

Random load factors can also be determined through spectral analysis. The computation of  $N_{rx}$  is described here for comparison with the Miles' approach. The transfer function of the system is first determined by applying a unit sinusoidal load in the X direction at each point where the NCPS interfaces with the rack structure. The load is applied through the DLOAD and RLOAD cards which in turn refers to DELAY, DPHASE, DAREA, and TABLED cards. For a constant phase, unit sinusoidal input, the DELAY and DPHASE cards may be omitted and a unit value specified for all frequencies on the TABLED card. The input acceleration spectrum (Fig. 3) is specified through RANDPS and TABRND cards. This random acceleration is enforced at all the interface points in the X direction using the large mass approach. A fictitious mass,  $M_a$ , about 1000 times larger than the existing grid point mass, is lumped at the interface X degree of freedom using a CMASS2 card and an equal value is specified in the

corresponding DAREA load card. This produces the desired acceleration which can be verified by plotting the acceleration spectrum at the input points and comparing it with Figure 3. If the agreement is not adequate, the value of  $M_a$  is increased until a good match is obtained.

The direct and modal solution techniques are used to carry out the response calculations. The relative efficiency of the two approaches depends on several factors including the number of modes retained in the analysis and the number of response frequencies. The selection of these parameters always represents a compromise between accuracy and efficiency. The frequencies chosen for response computations should have good resolution in the vicinity of system resonances in order to obtain reliable estimates of rms response. A total of 150 frequencies in the 0 to 500 Hz range are used for this analysis. The specification of damping properties in the direct and modal formulations are somewhat different. In the direct formulation, structural damping proportional to the stiffness matrix terms is specified both on the material data cards and as an overall uniform damping factor on the PARAM G data card. In the modal formulation, the damping factor is specified as a tabular function of frequency through the SDAMPING and TABDMP1 cards to represent the variation of structural damping for different modes. A damping factor of 0.1 is used for this analysis in order to be consistent with  $Q = 10$  used in Miles' relation.

The set of output points at which the response power spectrum and rms values are to be recovered must be chosen judiciously. The selection can be based on the same criteria used for choosing an ASET (analysis set of dynamic degrees of freedom); namely, the points should be uniformly dispersed throughout the structure and should include all large mass items. A set of 12 output points were chosen for the NCPS and the rms acceleration response at these points is computed (Table 2). When these values are averaged and the 3-sigma probability of occurrence criteria is applied, one obtains

$$N_{rx} = 10.67 \text{ g units} \quad (7)$$

This is almost 30% less than the value predicted using Miles' relation.

The rms response calculated using the direct and modal solution techniques is summarized in Table 2. For the same number of response frequencies, the modal solution required 350 seconds of CPU time with 20 modes included, 450 seconds with 40 modes, 600 seconds with 60 modes, whereas the direct solution required 13160 seconds. The response spectrum at selected output points on the model is shown in figures 4, 5, and 6. The solution obtained with 20 modes is clearly inadequate while the plots obtained using 60 modes and the direct approach are virtually indistinguishable. The spectra characteristically peak at 80 Hz which corresponds to the first X mode of vibration. The effects of higher modes can also be seen in the spectral plots.

## CONCLUDING REMARKS

An alternate method of estimating random vibration load factors for design and analysis of Spacelab payloads is presented. This method, based on spectral analysis, yields more refined random load estimates at the expense of being computationally more intensive than the Miles' approach. The computational effort can be reduced by using the modal formulation rather than the direct formulation for analysis. Significant reductions can be obtained in the random load estimates using this method. While the actual reduction depends upon the payload configuration being analyzed, reductions of 20 to 30% are typical. This method could be used to resolve difficult design problems owing to unreasonably high random load predictions by the Miles' relation.



## REFERENCES

1. Harris, C. M. and Crede, C. E., "Shock and Vibration Handbook," 2nd Edition, McGraw Hill Book Company, Inc., New York, 1976, pp.24-28.
2. Spacelab Payload Accomodation Handbook: Appendix B, SLP/2104-2, NASA Marshall Space Flight Center, October 1989.

TABLE I. EIGENVALUE ANALYSIS SUMMARY

MODE NO.	EIGENVALUE	RADIAN FREQUENCY	CYCLIC FREQUENCY
1	5.606682E+04	2.367843E+02	3.768539E+01
2	8.630755E+04	2.937815E+02	4.675677E+01
3	9.900016E+04	3.146429E+02	5.007697E+01
4	1.228704E+05	3.505287E+02	5.578837E+01
5	1.483150E+05	3.851169E+02	6.129326E+01
6	1.896878E+05	4.355316E+02	6.931701E+01
7	2.301971E+05	4.797886E+02	7.636072E+01
8	2.535269E+05	5.035146E+02	8.013683E+01
9	3.989269E+05	6.316066E+02	1.005233E+02
10	4.230480E+05	6.504214E+02	1.035178E+02
11	4.433938E+05	6.658782E+02	1.059778E+02
12	4.683294E+05	6.846459E+02	1.089170E+02
13	5.100052E+05	7.141465E+02	1.136599E+02
14	5.928444E+05	7.699639E+02	1.225435E+02
15	7.732380E+05	8.793395E+02	1.399512E+02
16	7.972504E+05	8.928888E+02	1.421077E+02
17	8.693909E+05	9.324113E+02	1.483979E+02
18	9.891926E+05	9.945816E+02	1.582926E+02
19	1.104014E+06	1.050721E+03	1.672274E+02
20	1.231224E+06	1.109605E+03	1.765992E+02
21	1.492994E+06	1.221881E+03	1.944685E+02

TABLE II RMS ACCELERATION RESPONSE SUMMARY

GRID POINT	LOCATION	MODAL SOLUTION			DIRECT SOLUTION (g)
		20 MODES (g)	40 MODES (g)	60 MODES (g)	
5059	Rear Left Side Panel	2.07	3.50	3.52	3.52
5080	Rear Right Side Panel	2.06	3.97	3.99	3.98
5056	Front Left Side Panel	2.47	2.47	2.49	2.50
5077	Front Right Side Panel	2.47	2.47	2.49	2.50
5076	Top Left Rear Panel	2.08	4.53	4.58	4.58
5073	Top Left Front Panel	2.24	2.48	2.87	2.88
5094	Top Right Front Panel	2.24	2.48	2.87	2.88
5097	Top Right Rear Panel	2.08	4.89	4.91	4.91
5193	M/C Unit	2.07	4.33	4.46	4.46
5204	CPU Unit	2.81	3.89	3.95	3.95
5205	M/C Mount	2.12	4.03	4.04	4.03
5042	Front Panel	2.47	2.47	2.49	2.50
Average		2.26	3.46	3.55	3.56

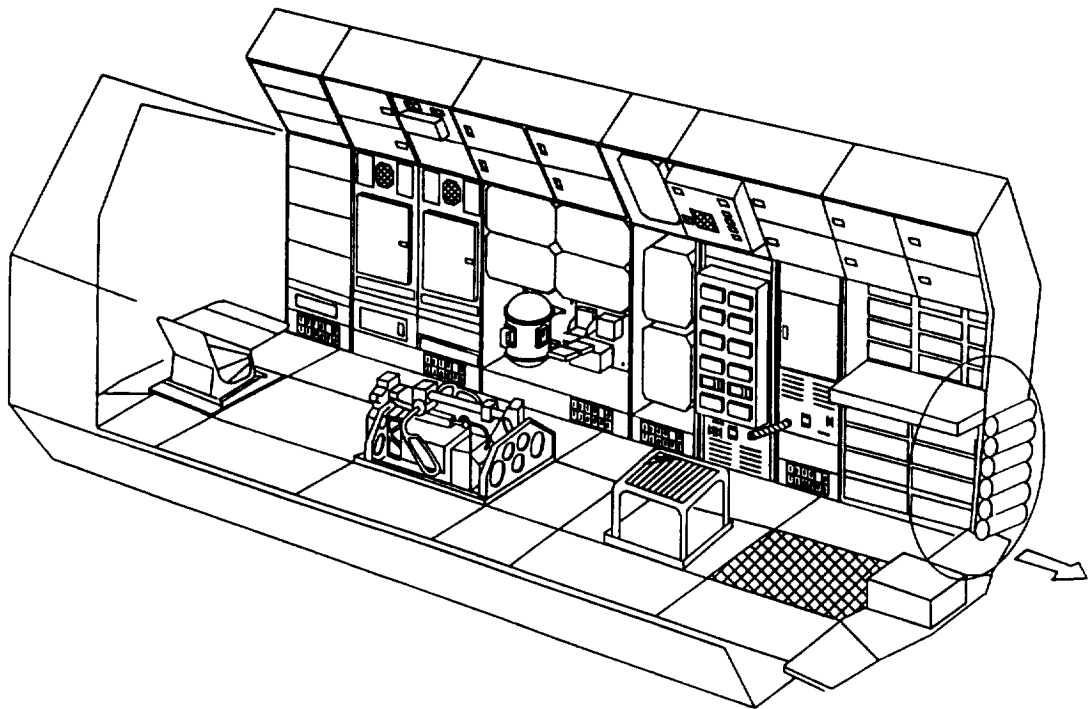
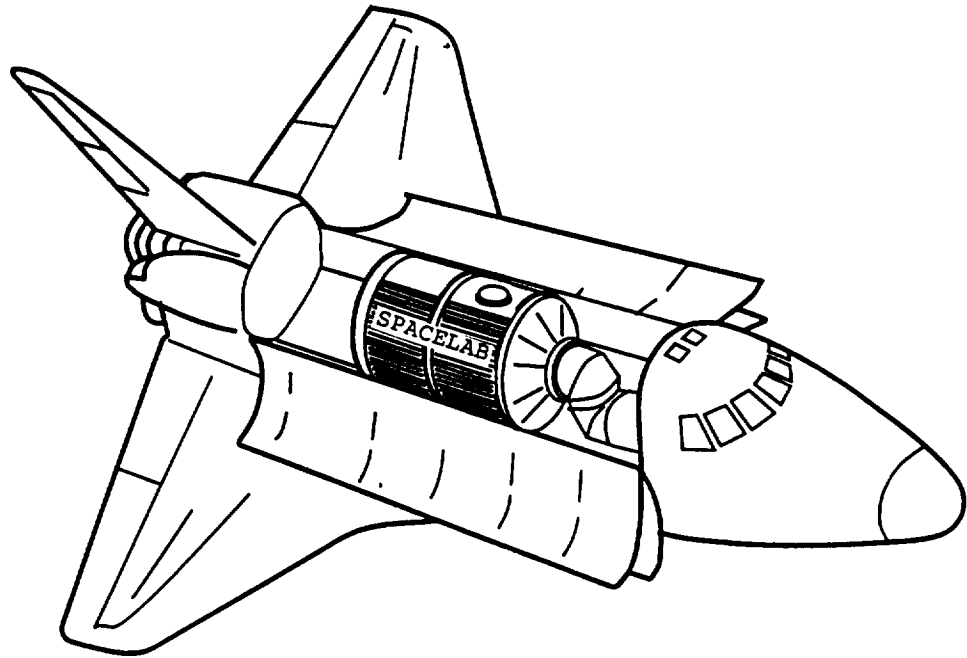


FIG. 1. TYPICAL SPACELAB CONFIGURATION

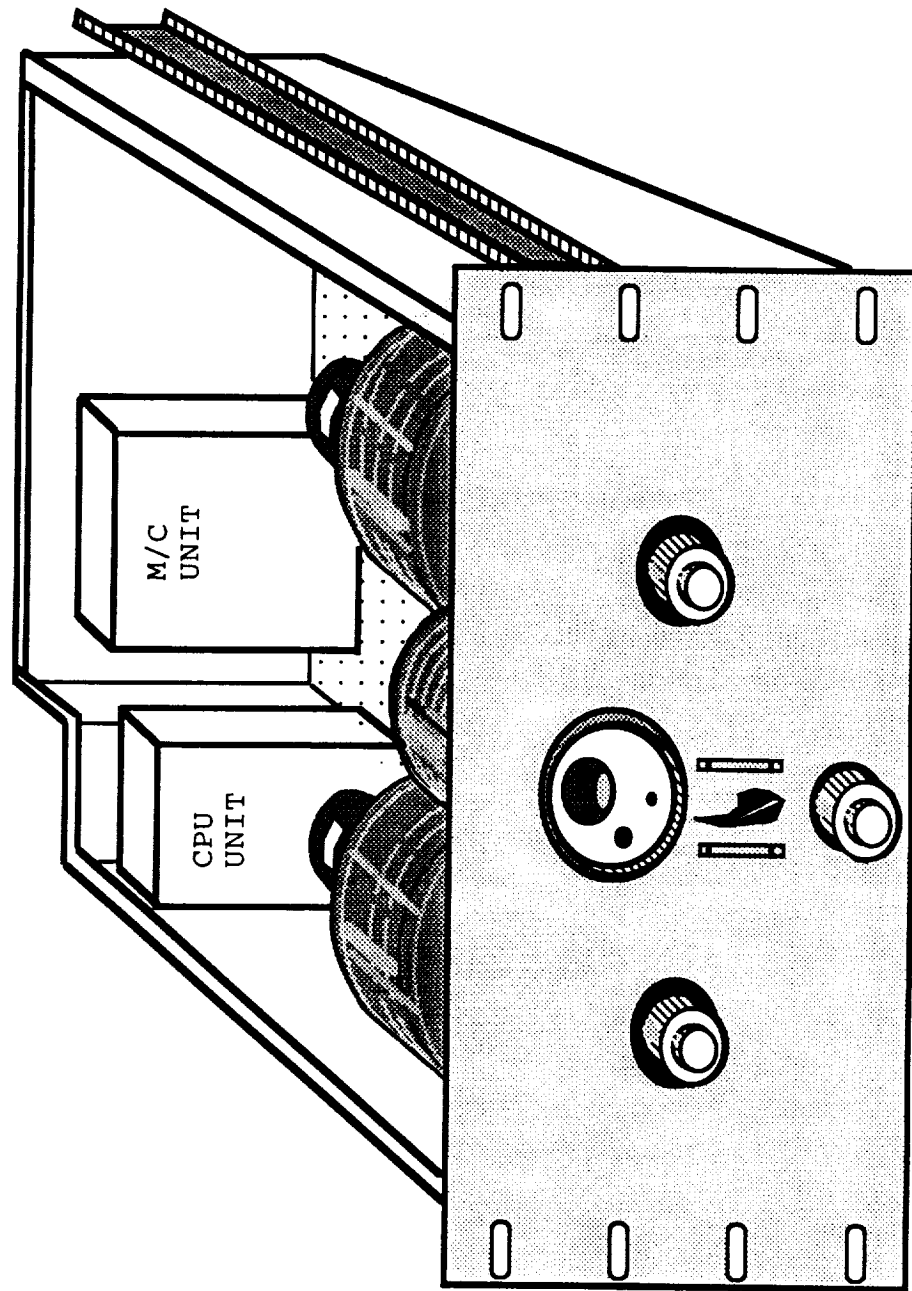


FIG. 2. SCHEMATIC OF THE NECK CHAMBER PRESSURE SYSTEM ASSEMBLY

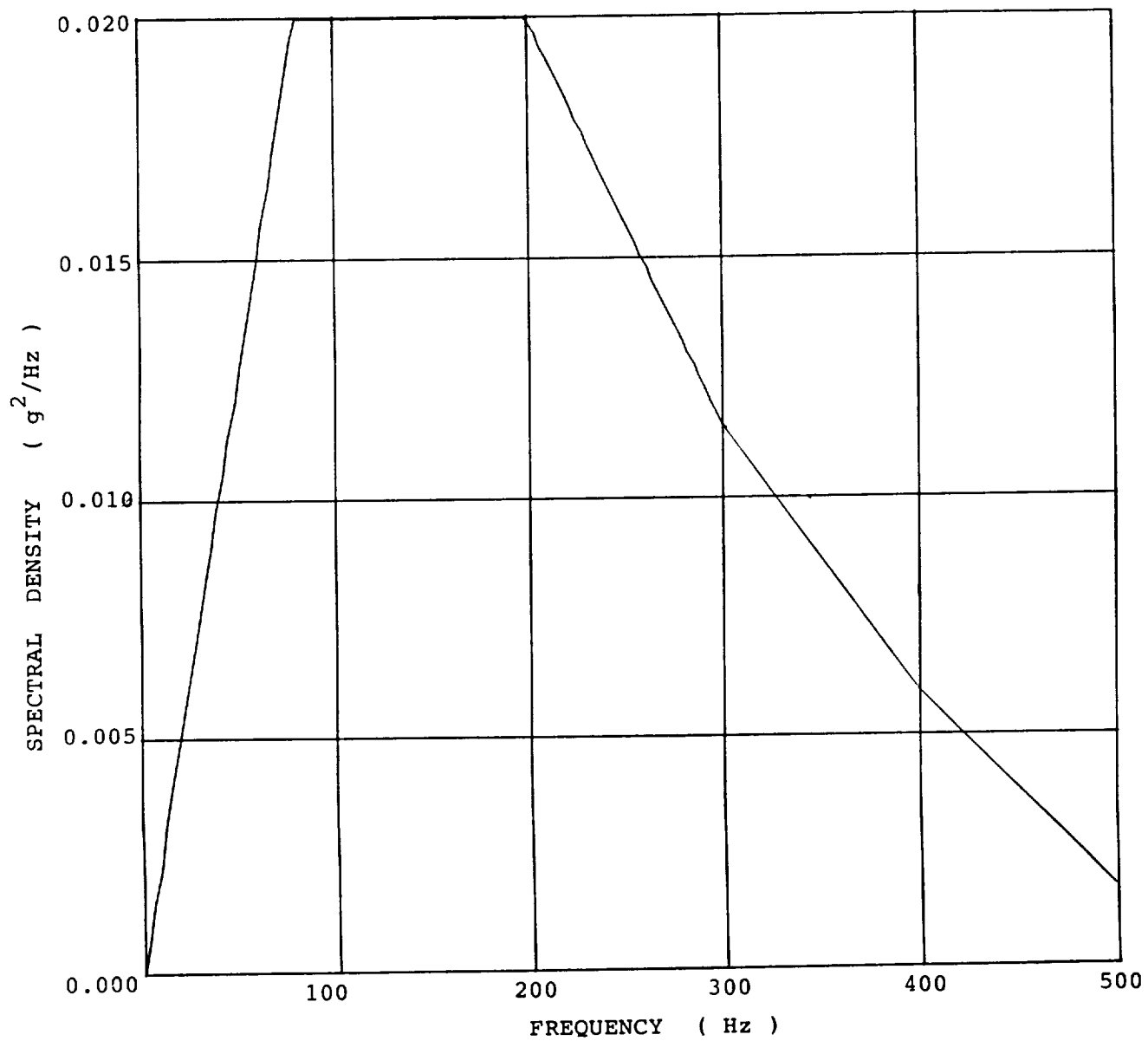


FIG. 3. INPUT ACCELERATION SPECTRUM

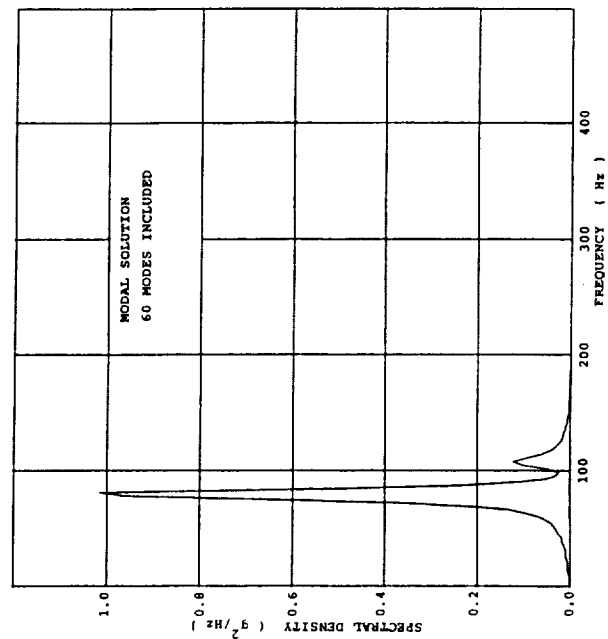
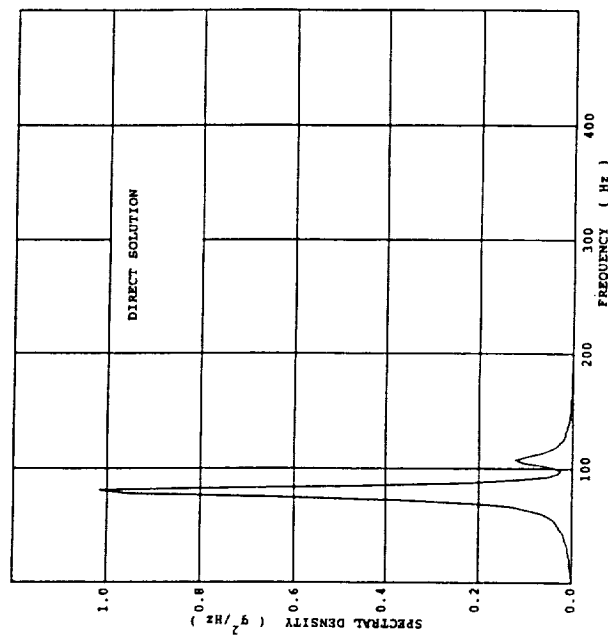
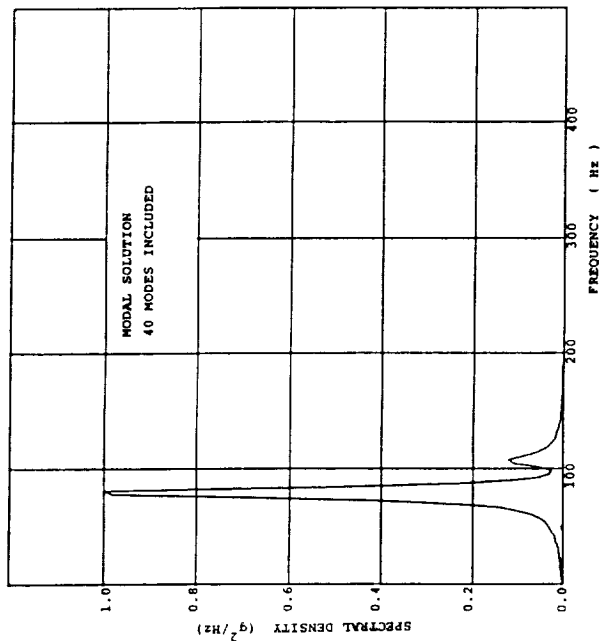
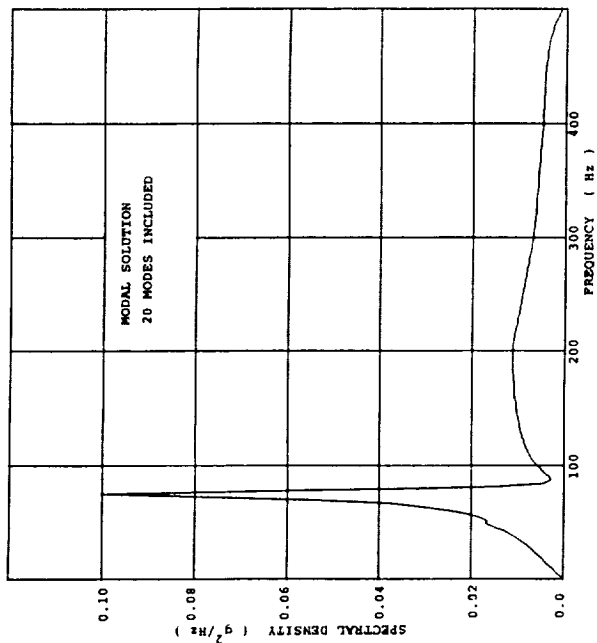


FIG. 4. ACCELERATION RESPONSE SPECTRUM AT GRID POINT 5073

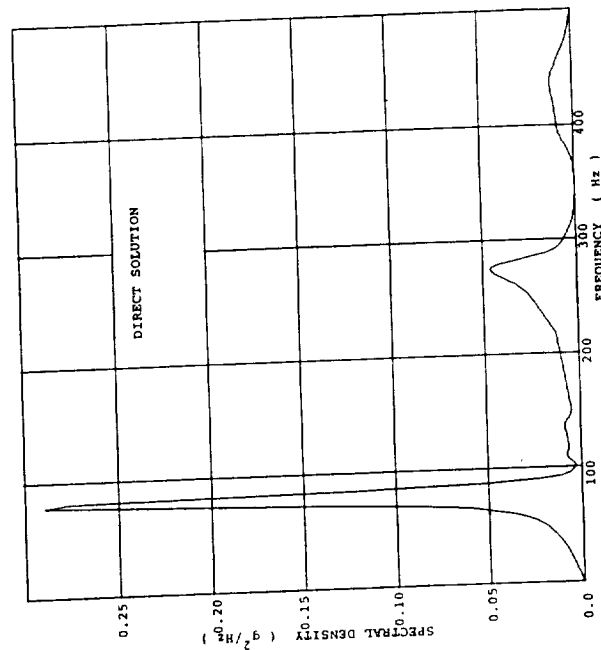
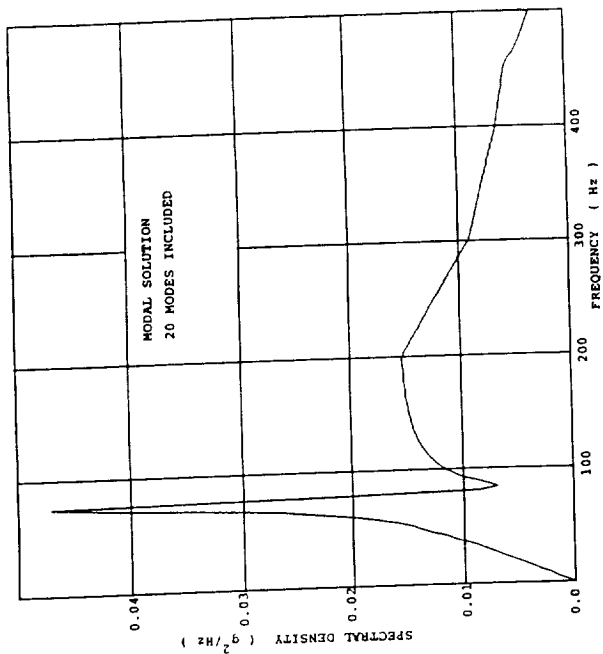
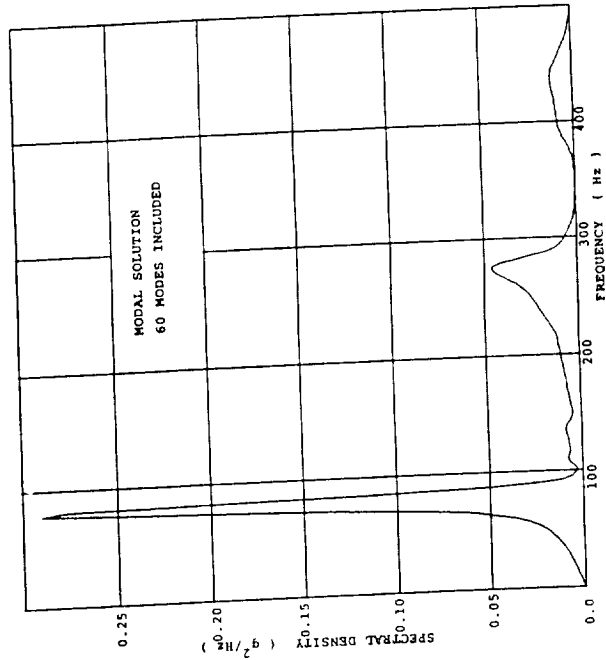
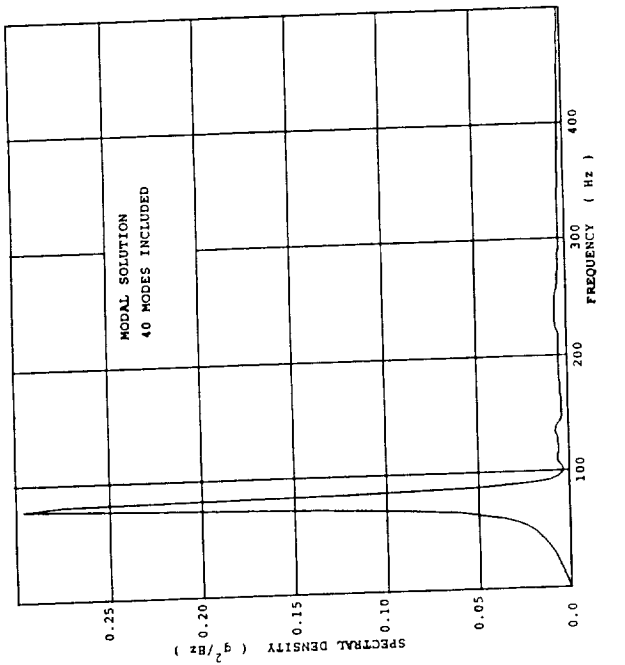


FIG. 5. ACCELERATION RESPONSE SPECTRUM AT GRID POINT 5205



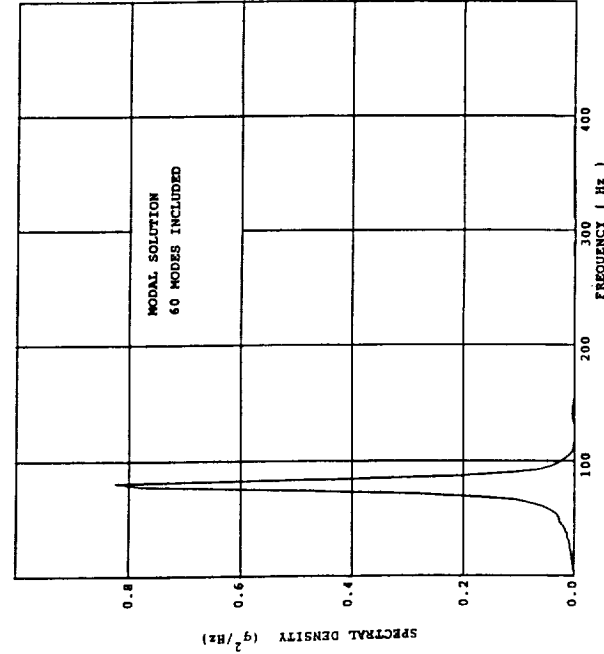
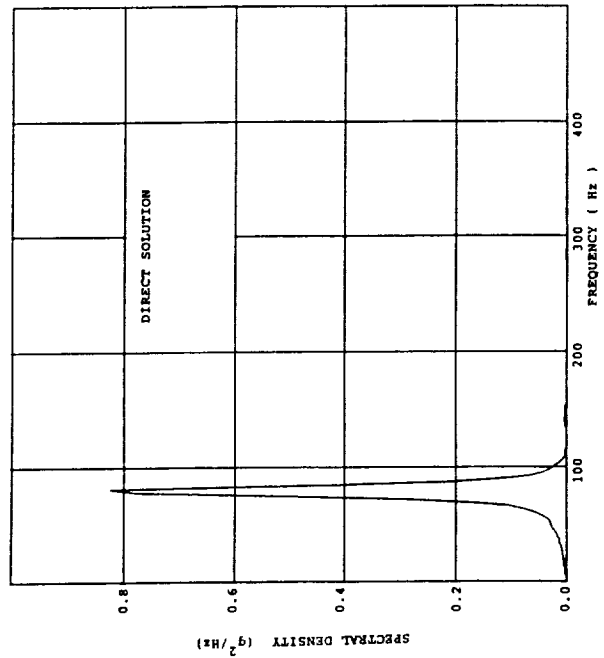
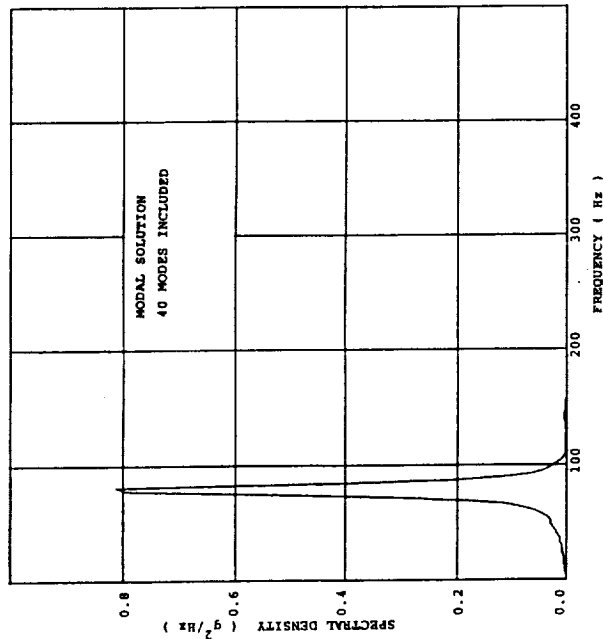
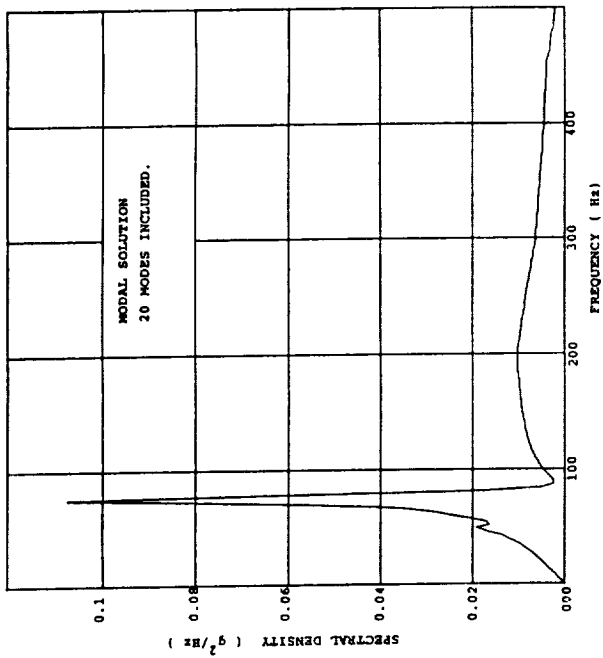


FIG. 6. ACCELERATION RESPONSE SPECTRUM AT GRID POINT 5059